GCSE Mathematics (1MA1) – Higher Tier Paper 3H (Set 2)

Aiming for Grade 9 – Spring 2022 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

NOTES ON MARKING PRINCIPLES

Guidance on the use of codes within this mark scheme

M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.

P1 – process mark. This mark is generally given for setting up an appropriate process to find a solution in the context of the question.

A1 – accuracy mark. This mark is generally given for a correct answer following correct working.

B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.

C1 – communication mark. This mark is given for explaining your answer or giving a conclusion in context supported by your working.

Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$h = 5 \tan 40^{\circ}$	P1	This mark is given for a process to find the height of the triangle
	= 4.195	P1	This mark is given for finding the height of the triangle
	$(\frac{1}{2} \times 10 \times 4.195) + (10 \times 12) = 140.975$	P1	This mark is given for a process to find the area of the cross-section of the prism
	140.975 × 20	P1	This mark is given for a process to find the volume of the cross-section of the prism
	2820 (to 3 significant figures)	A1	This mark is given for a correct answer only

Question 2 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Assume Olivia and Jessica have $(9 + 1)a$ sweets shared in ratio $9a : a$	P1	This mark is given for a process to find how many sweets Olivia and Jessica have
	Then Fran and Gary have $20a$ sweets shared in ratio 2: $3 = 8a : 12a$	P1	This mark is given for a process to find how many sweets Fran and Gary have
	w: x: y: z are in ratio $8a: 12a: 9a: a$	P1	This mark is given for a process to find an unsimplified ratio
	8:12:9:1	Al	This mark is given for the correct answer only

Question 3 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\angle DBF = 180 - 100 = 80$ Opposite angles of a cyclic quadrilateral add up to 180°	M1	This mark is given for a method to find the size of $\angle DBF$
	$\angle BFD = 180 - 80 - 40 = 60$ Angles in a triangle add up to 180	M1	This mark is given for a method to find the size of $\angle BFD$
	$\angle ABD = 60$ Alternate segment theorem	A1	This mark is given for the correct answer only
		C1	This mark is given for all three reasons stated

Part	Working an or answer examiner might expect to see	Mark	Notes
(a)	y = x + 3 y = x + 3 x = 2 y = x + 3 y	M2 M1 A1	These marks are given for a method to draw 3 of lines $x = 2$, $y = x + 3$, $2x + 3y = 6$ correctly (M1 is given for 2 lines drawn correctly) This mark is given for region a region <i>R</i> which satisfies at least 2 of the inequalities $x \le 2$, $y \le x + 3$ and $2x + 3y$ ≥ 6 This mark is given for a fully correct graph
(b)	Geoffrey in incorrect; 4 < 8, 4 > 1 and $2 + 4 = 6$ so $(2, 4)$ satisfies all the inequalities	B1	This mark is given for a valid conclusion with a correct reason given

Question 4 (Total 5 marks)

Question 5 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Let $10x = 7.333$ Then $100x = 73.333$ 90x = 66	M1	This mark is given for a method to find an equation without a recurring decimal
	$x = \frac{66}{90} = \frac{11}{15}$	A1	This mark is given for showing the recurring decimal in as a rational number

Question 6 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$(7.2 - 6.4) \times 10 = 8$ $(7.6 - 7.2) \times 50 = 20$	P1	This mark is given for a process to find at least one frequency
	$(8.0 - 7.6) \times 100 = 40$ $(8.2 - 8.0) \times 60 = 12$	P1	This mark is given for a process to find at all frequencies
	$\frac{(6.8 \times 8) + (7.4 \times 20) + (7.8 \times 40) + (8.1 \times 12)}{8 + 20 + 40 + 12}$ $= \frac{6116}{22}$	P1	This mark is given for a process to find an estimate of the mean
	80 7.645	A1	This mark is given for the correct answer only

Question 7 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\frac{6x^3}{(9x^2 - 144)} \times \frac{3(x - 4)}{2x^4}$	M1	This mark is given for a method to invert the fraction and multiply
	$\frac{6x^3}{3(x+4)\times 3(x-4)} \times \frac{3(x-4)}{2x^4}$	M1	This mark is given for a method to simplify the fraction
	$\frac{6x^{3}}{3(x+4)\times 3(x-4)} \times \frac{3(x-4)}{2x^{4}}$ $= \frac{6x^{3}}{(x+4)\times 6x^{4}}$		
	$\frac{1}{x(x+4)}$	Al	This mark is given for the correct answer only

Part	Working an or answer examiner might	Mark	Notes
(a)	expect to see	M1	This mark is given for stating an upper or lower bound for <i>l</i> or <i>T</i>
	$\frac{1}{2}((4+6) + (6+7.2) + (7.2+7.8))$	M1	This mark is given for a complete method to find an estimate for the area under the graph
	19.1	A1	This mark is given for a correct answer in the range 19 to 20
(b)	Distance travelled by the object	C1	This mark is given for a correct statement

Question 8 (Total 4 marks)

Question 9 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$x^{\frac{6}{n}} = x^3, \frac{6}{n} = 3$	P1	This mark is given for a method to find the product of two linear expressions
	$a^{\frac{1}{n}} = 7$		
	n = 2 When $n = 2, a = 49$	A1	This mark is given for two correct answers only

Question 10 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{30 \times 9}{2}$	M1	This mark is given for a method to find the area of a triangle with coordinates (0, 0), (30, 0) and $(30, 9)$
	135	A1	This mark is given for a correct estimate of the distance the car travelled
(b)	underestimate, since the area between the line and the curve is not included	C1	This mark is given for a valid conclusion and reason given
(c)	For example: Julian's method gives average speed in first 60 seconds Julian has not drawn a tangent at time 60 seconds	C1	This mark is given for a correct explanation
	Julian has not worked out the gradient at time 60 seconds		

Question 11 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$x = ky^2$ $y = cz^3$	M1	This mark is given for setting up an equation of proportionality
	$x = k(cz^3)^2$ $x = kc^2z^6$	M1	This mark is given for setting up an equation of proportionality eliminating <i>y</i>
	When $z = 2$, $x = kc^2 \times 64$ Constant $kc^2 = \frac{1}{2}$	M1	This mark is given for substituting $z = 2$ and $x = 32$ to find the value of the constant
	$x = \frac{1}{2}z^{6}$	A1	This mark is given for a correct answer only

Question 12 (Total 5 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
(a)	5c + d = c + 4d $4c = 3d$	P1	This mark is given for process to isolate terms in <i>c</i> to obtain $4c = 3d$
	3:4	A1	This mark is given for the correct answer only
(b)	$6x^2 - 7xy - 20y^2 = 0$	P1	This mark is given for a process to form a quadratic equation equal to 0
	(2x - 5y)(3x + 4y) = 0	P1	This mark is given for a process to factorise the quadratic equation
	5:2	A1	This mark is given for the correct answer only

Question 13 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$x^2 + 3x - 3 = 5x - 4$	M1	This mark is given for a method to eliminate y
	$x^{2} - 2x + 1 = 0$ (x - 1)(x - 1) = 0 x = 1		This mark is given for a method to rearrange to form a quadratic equation
			This mark is given for correctly factorising and solving the quadratic equation
	There is only one value of <i>x</i> , so only one point (set of coordinates) in common	C1	This mark is given for a correct answer only

Working or answer an examiner might Part Mark Notes expect to see This mark is given for a tangent to the M1 50 curve drawn at t = 1240 30 Volume (litres) 20 10 0 20 30 Time (t seconds) This mark is given for a method to find For example: M1 the gradient of the tangent $28 \div 30$ This mark is given for an answer in the 0.933 A1 range 0.7 to 1.0

Question 14 (Total 3 marks)

Question 15 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Lower bound for $AH = 11.25$	M1	This mark is given for a method to find the lower bound for the length AH
	Let $a =$ the length of one side of the cube $a^2 + a^2 + a^2 = 11.25^2$	M1	This mark is given for a method to use $a^2 + a^2 + a^2$ as the length of the diagonal
	$a^{2} + a^{2} + a^{2} = 11.25^{2}$ $3a^{2} = 11.25^{2}$	M1	This mark is given for a method to write an equation to find the length of a side
	<i>a</i> = 6.5	A1	This mark is given for the correct answer in the range $6.49 - 6.50$

Question 16 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\overrightarrow{DB} = \frac{3}{5}OB$	P1	This mark is given for a process to find a relationship involving \overrightarrow{DB}
$\overrightarrow{DB} = \frac{3}{5}(\mathbf{a} + \mathbf{b})$		P1	This mark is given for a process to find a vector expression for \overrightarrow{DB}
	$\overrightarrow{BE} = \frac{1}{5} \overrightarrow{BC} = \frac{1}{5} (-\mathbf{b} - \mathbf{a} + 3\mathbf{b})$	P1	This mark is given for a process to find a vector expression for \overrightarrow{BE}
	$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE} = \frac{2}{5}\mathbf{a} + \mathbf{b}$	A1	This mark is given for a correct answer only

Question 17 (Total 3 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	Population in $2017 = 4000$ Population in $2018 = 4000k$ Population in $2019 = 4000k^2 = 3160$	P1	This mark is given for a process to use the formula for the population
	$k^2 = \frac{3160}{4000}, k = \sqrt{\frac{3160}{4000}}$	P1	This mark is given for a process to find <i>k</i>
	0.95	A1	This mark is given for a correct answer only

Question 18 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	l+l-w=11-3	P1	This mark is given for a process to set up and equation for the width of the pattern
	l+l+w=20-4	P1	This mark is given for a process to set up and equation for the height of the pattern
	l + l - w = 8 l + l + w = 16 4l = 24 so $l = 6$ and $w = 4$	P1	This mark is given for a process to find the length and width of the rectangles used in the pattern
	(3+6,4+4)	P1	This mark is given for a process to find the coordinates of <i>C</i>
	(9, 8)	A1	This mark is given for the correct answer only

Question 19 (Total 1 mark)

Part	Working or answer an examiner might expect to see		Notes
	98 ⁹¹	A1	This mark is given for a correct answer only

Question 20 (Total 4 marks)

Working or answer an examiner might expect to see	Mark	Notes
Area $ABCDEF = 6 \times \frac{1}{2} \times x \times x \times \sin 60$	P1	This mark is given for a start to the process to find the area of <i>ABCDEF</i>
$= \frac{6\sqrt{3}}{4}x^2 = \frac{3\sqrt{3}}{2}x^2$	P1	This mark is given for a full process to find the area of <i>ABCDEF</i>
Area $FGHIJK = 6 \times \frac{1}{2} \times px \times px \times \sin 60$	P1	This mark is given for a process to find the area of <i>FGHIJK</i>
$= \frac{6\sqrt{3}}{4}p^2x^2 = \frac{3\sqrt{3}}{2}p^2x^2$		
The shaded region of the diagram is $\frac{3\sqrt{3}}{2}p^2x^2 - \frac{3\sqrt{3}}{2}x^2 = \frac{3\sqrt{3}}{2}(p^2 - 1)x^2$	C1	This mark is given for fully correct algebra to show the result required
	expect to see Area ABCDEF = $6 \times \frac{1}{2} \times x \times x \times \sin 60$ $= \frac{6\sqrt{3}}{4}x^2 = \frac{3\sqrt{3}}{2}x^2$ Area FGHIJK = $6 \times \frac{1}{2} \times px \times px \times \sin 60$ $= \frac{6\sqrt{3}}{4}p^2x^2 = \frac{3\sqrt{3}}{2}p^2x^2$ The shaded region of the diagram is	expect to seeArea ABCDEF = $6 \times \frac{1}{2} \times x \times x \times \sin 60$ P1 $= \frac{6\sqrt{3}}{4}x^2 = \frac{3\sqrt{3}}{2}x^2$ P1Area FGHIJK = $6 \times \frac{1}{2} \times px \times px \times \sin 60$ P1 $= \frac{6\sqrt{3}}{4}p^2x^2 = \frac{3\sqrt{3}}{2}p^2x^2$ P1The shaded region of the diagram isC1

Question 21 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\left(\frac{1}{2}\right)^n$	M1	This mark is given for a finding the probability that Pat throws <i>n</i> heads or <i>n</i> tails
	$1 - \left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n$	A1	This mark is given for the correct answer only

Question 22 (Total 4 marks)

Part	Working an or answer examiner might expect to see	Mark	Notes
	For triangle ABD, area = $\frac{1}{2} \times AB \times AD \times \sin x$ For triangle ADC, area = $\frac{1}{2} \times AD \times AC \times \sin x$	M1	This mark is given for a method to use area = $\frac{1}{2}ab \sin C$ to find the areas of <i>ABD</i> and <i>ADC</i>
	For triangle <i>ABD</i> , area = $\frac{1}{2} \times h \times BD$ For triangle <i>ADC</i> , area = $\frac{1}{2} \times h \times DC$	M1	This mark is given for a method to find another expression for the areas of <i>ABD</i> and <i>ADC</i>
	$\frac{0.5 \times AB \times AD \times \sin x}{0.5 \times AD \times AC \times \sin x} = \frac{0.5 \times h \times BD}{0.5 \times h \times DC}$	M1	This mark is given for a method to find the ratio of the areas of the triangles
	$\frac{AB}{AC} = \frac{BD}{DC} \text{ thus } \frac{AB}{BD} = \frac{AC}{DC}$	C1	This mark is given for a full method to arrive at the given answer

				Edexcel	averages:	scores of	candidate	es who acl	nieved gra	de:
Qn	Skill tested	Max score	Mean score %	ALL	9	8	7	6	5	4
1	Volume cuboids and other right prisms (including cylinders)	5	36	1.81	4.73	4.09	3.02	2.01	1.14	0.55
2	Multiplicative relationship between two quantities	4	29	1.17	3.10	2.17	1.98	1.44	1.00	0.68
3	Circle theorems	4	29	1.15	3.51	2.99	2.12	1.30	0.55	0.16
4	Solve linear inequalities	5	28	1.40	4.04	3.81	2.50	1.39	0.68	0.36
5	Recurring decimals and their corresponding fractions	2	26	0.51	1.55	1.30	0.92	0.60	0.26	0.08
6	Histograms with equal and unequal class intervals	4	23	0.93	3.13	2.43	1.70	0.96	0.49	0.11
7	Simplify and manipulate algebraic expressions and fractions	3	23	0.69	2.55	1.91	1.43	0.83	0.54	0.16
8	Distance-time graphs, velocity-time graphs	4	22	0.87	3.50	2.55	1.93	1.03	0.53	0.16
9	Roots and powers	2	21	0.41	1.66	1.23	0.72	0.43	0.25	0.16
10	Distance-time graphs, velocity-time graphs	4	19	0.75	3.30	2.49	1.36	0.67	0.21	0.06
11	Solve problems involving direct and inverse proportion	4	18	0.70	3.04	1.97	1.33	0.92	0.48	0.14
12	Multiplicative relationship between two quantities	5	15	0.75	2.41	1.41	1.09	0.85	0.57	0.28
13	Translate situations or procedures into algebraic expressions, formulae or equations	4	15	0.59	3.61	2.34	1.23	0.48	0.25	0.06
14	Gradient at a point on a curve as the instantaneous rate of change	3	14	0.43	2.51	1.54	0.98	0.46	0.12	0.04
15	Limits of accuracy; bounds	4	14	0.54	2.52	1.16	0.77	0.48	0.35	0.17
16	Vectors	4	13	0.51	3.62	2.43	1.13	0.36	0.06	0.01
17	Growth and decay, compound interest	3	11	0.33	2.72	1.44	0.52	0.21	0.09	0.03
18	Translate situations or procedures into algebraic expressions, formulae or equations	5	9	0.46	4.21	2.03	0.97	0.21	0.05	0.00
19	Primes, factors, multiples	1	8	0.08	0.46	0.21	0.12	0.05	0.03	0.02
20	Mensuration and calculation	4	3	0.10	1.47	0.34	0.06	0.01	0.00	0.00
21	Probabilities of an exhaustive set of outcomes	2	2	0.03	0.38	0.09	0.05	0.02	0.00	0.00
22	Sine and cosine rule	4	1	0.03	0.54	0.13	0.01	0.01	0.00	0.00
		80	18	14.24	58.56	40.06	25.94	14.72	7.65	3.23

Students who were awarded a Grade 9 averaged 58 marks on this set of questions in the November 2020 and 2021 examinations. That said, there weren't very many of them in these cohorts.

Aiming for 9 – Set 2 (Spring 2022)

Suggested grade boundaries

	Max	9	8	7	6	5	4
1H	80	56	39	24	14	7	3
2H	80	54	37	23	12	6	2
3H	80	49	33	20	11	5	2
Total	240	159	109	67	37	18	7

Grade boundaries are based on the average performance data for students answering these questions who gained grades 4-9 in the November 2020 & 2021 GCSE Mathematics examinations at Higher tier.

Students did not answer these questions as 90-minute tests, of course; so there is some scope for adjustment. These boundaries are for guidance only.